Linear-Programming Approximations of AC Power Flows

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Outline

• Motivation
• The LPAC Models
• Experimental Results
  • LDC versus LPAC versus AC solutions
  • LPAC Variants
• Capacitor Placement Problem
• Power Restoration
Motivation

Collaboration with LANL
Motivation
Power Restoration

- One challenge (PSCC’11)
  - Schedule a fleet of repair crews to repair the grid and minimize the overall size of the blackout after a disaster

- Two fundamental aspects
  - Scheduling the repairs
  - Scheduling the power restoration
  - Both are challenging in their own right

- Assumptions for Last-Mile Restoration
  - Steady state behavior of the power grid
  - Ability to shed load and generation continuously
  - Transient/configuration aspects in a second step
Power Restoration

Restoration Timeline

Minimize

Increase in served demand

Component repair

Power Flow

Time
Power Restoration

- **Optimal Activation Problem**
  - Generalized optimal line switching [Fisher et al, 98]
- **Approximate the power flows equations**
  - Linear DC Model
- **Discrete optimization over the LDC model**
  - MIP solver
- **Solutions to large benchmarks [CPAIOR’12]**
  - 4000 components, a third of which were damaged
  - Using hybrid optimization (MIP + CP + LNS)
Optimal Activation

- find which items to activate
- find how much power to produce and consume
- find the phase angles at buses
- to maximize the served load

- Generalized optimal line switching [Fisher et al, 98]

\[
\begin{align*}
\text{Inputs:} & \quad \mathcal{P} \mathcal{N} = (N, L) \quad \text{the power network} \\
& \quad D \quad \text{the set of damaged items} \\
& \quad R \quad \text{the set of repaired items} \\
\text{Variables:} & \quad y_i \in \{0, 1\} \quad \text{- item } i \text{ is activated} \\
& \quad z_i \in \{0, 1\} \quad \text{- item } i \text{ is operational} \\
& \quad P_i^l \in (-P_i^l, P_i^l) \quad \text{- power flow on line } i \\
& \quad P_i^n \in (0, P_i^n) \quad \text{- power flow on node } i \\
& \quad \theta_i \in (-\frac{\pi}{2}, \frac{\pi}{2}) \quad \text{- phase angle on bus } i \\
\text{Maximize} & \quad \sum_{b \in N^b} \sum_{i \in N_i^b} P_i^n \\
\text{Subject to:} & \quad y_i = 1 \quad \forall i \in N \setminus D \quad (2) \\
& \quad y_i = 0 \quad \forall i \in D \setminus R \quad (3) \\
& \quad z_i = y_i \quad \forall i \in N^b \quad (4) \\
& \quad z_i = y_i \wedge y_j \quad \forall j \in N^b, \forall i \in N_i^b \quad (5) \\
& \quad z_i = y_i \wedge y_{L_i^+} \wedge y_{L_i^-} \quad \forall i \in L \quad (6) \\
& \quad \sum_{j \in N_i^b} P_j^n = \sum_{j \in N_i^b} P_j^n + \sum_{j \in L_i^+} P_j^l - \sum_{j \in L_i^-} P_j^l \quad \forall i \in N^b \quad (7) \\
& \quad 0 \leq P_i^n \leq P_i^l \ast z_i \quad \forall i \in N_i^b \cup N_i^j \quad (8) \\
& \quad -P_i^l \ast z_i \leq P_i^l \leq P_i^l \ast z_i \quad \forall i \in L \quad (9) \\
& \quad P_i^l \geq B_i \ast (\theta_{L_i^+} - \theta_{L_i^-}) + M \ast (\neg z_i) \quad \forall i \in L \quad (10) \\
& \quad P_i^l \leq B_i \ast (\theta_{L_i^+} - \theta_{L_i^-}) - M \ast (\neg z_i) \quad \forall i \in L \quad (11) \\
\end{align*}
\]

Figure 1: A MIP Model for the Unserved Load.
Power Restoration

Restoration Timeline

Power Flow

Time

Relaxation
- MIP
- LNS
- Baseline
A fundamental open question

- Is this “optimal” restoration plan “feasible” operationally?

- These are not normal operating conditions
  - “Maddeningly difficult” to find an AC solution in cold start contexts [Overbye et al, 2004]
- The network is stressed
  - Does the LDC model “overfit”? 
- How accurate is the LDC model?
  - Can the LDC solution be turned into an AC solution?
N-3 Contingencies (IEEE-30)

Line Apparent Power Correlation (MVA)

- Small Line Phase Angle
- Large Line Phase Angle

AC Power Flow

LDC Power Flow

IEEE PES'12
N-3 Contingencies

Bus Phase Angle Correlation (rad)

- LDC Power Flow vs AC Power Flow
- Black circles: Small Line Phase Angle
- Red triangles: Large Line Phase Angle
N-3 Contingencies

Line Reactive Power Density

- Small Line Phase Angle
- Large Line Phase Angle

Line Count (log scale)

AC Reactive Power Flow (MVar)

-145 -105 -75 -45 -15 5 25 45 65 85

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From imagination to impact
Power Restoration

DC Restoration Timeline

Restoration Action

DC Power Flow (MW)

LDC–ROP
Power Restoration

AC Restoration Timeline

AC Power Flow (MW)

Restoration Action

HELP WANTED

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• Find an approximation of AC power flows that
  • is more accurate than the LDC model
  • is useful outside normal operating conditions
  • reasons about voltage magnitudes and reactive power
  • can be embedded in discrete optimization solvers
    • mixed integer programming solvers

• Applications
  • Power restoration, vulnerability analysis, capacitor placement, expansion planning, …
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AC Power Flow

\[ p_n = \sum_{m \neq n} p_{nm} \]

\[ q_n = \sum_{m \neq n} q_{nm} \]

\[ p_{nm} = |\tilde{V}_n|^2 g_{nm} - |\tilde{V}_n||\tilde{V}_m| g_{nm} \cos(\theta_n^\circ - \theta_m^\circ) - |\tilde{V}_n||\tilde{V}_m| b_{nm} \sin(\theta_n^\circ - \theta_m^\circ) \]

\[ q_{nm} = -|\tilde{V}_n|^2 b_{nm} + |\tilde{V}_n||\tilde{V}_m| b_{nm} \cos(\theta_n^\circ - \theta_m^\circ) - |\tilde{V}_n||\tilde{V}_m| g_{nm} \sin(\theta_n^\circ - \theta_m^\circ) \]
Linear Programming Approximations

• **Hot-Start Context**
  - An AC base-point solution is available

• **Warm-Start Context**
  - Target voltage magnitudes are available and “useful”
  - E.g., from normal operating conditions

• **Cold-Start Context**
  - No useful information is available on voltage magnitudes
Hot-Start LP Approximation

\[
\begin{align*}
\hat{p}_{nm}^h &= |\tilde{V}_n^h|^2 g_{nm} - |\tilde{V}_n^h||\tilde{V}_m^h|g_{nm}\cos(\theta_n^o - \theta_m^o) - |\tilde{V}_n^h||\tilde{V}_m^h|b_{nm}(\theta_n^o - \theta_m^o) \\
\hat{q}_{nm}^h &= -|\tilde{V}_n^h|^2 b_{nm} + |\tilde{V}_n^h||\tilde{V}_m^h|b_{nm}\cos(\theta_n^o - \theta_m^o) - |\tilde{V}_n^h||\tilde{V}_m^h|g_{nm}(\theta_n^o - \theta_m^o)
\end{align*}
\]

- Two approximations
  - \(\sin(x)\) is approximated by \(x\)
  - piecewise approximation of \(\cos(x)\)
Hot-Start LP Approximation

Fig. 1. A Piecewise-Linear Approximation of Cosine using 7 Inequalities.
Warm-Start LP Approximation

• Understanding power flows [Grainger, 94]
  • Phase angle differences determine active power
  • Voltage magnitude differences determine reactive power

• Experiments
  • Per unit system
  • Look at how the equations behave when
    • $g = 0.2$ and $b = 1.0$
    $$|\vec{V}_n| = 1.0, |\vec{V}_m| \in (1.2, 0.8), \theta_n - \theta_m \in (-\pi/6, \pi/6)$$
Warm-Start LP Approximation

Active Power Field

Reactive Power Field

Voltage Difference

Angle Difference (rad)

Voltage Difference

Angle Difference (rad)
Warm-Start LP Approximation

• Assumptions
  • We have target voltages

• Basic approach
  • Active power as in the hot-start model
  • Reactive power should capture voltage magnitudes and phase angles

• Key idea
  • Substitute $|\tilde{V}| = |\tilde{V}^t| + \phi$ into the power flow equations
Warm-Start LP Approximation

- Reactive power

\[ q_{nm} = q_{nm}^t + q_{nm}^\Delta \]

- Target part

\[ q_{nm}^t = -|\tilde{V}_n|^2 b_{nm} + |\tilde{V}_n| |\tilde{V}_m| b_{nm} \cos(\theta_n^\circ - \theta_m^\circ) - |\tilde{V}_n| |\tilde{V}_m| g_{nm} \sin(\theta_n^\circ - \theta_m^\circ) \]

- Delta part

\[ q_{nm}^\Delta = -(2|\tilde{V}_n| \phi_n + \phi_n^2) b_{nm} - (|\tilde{V}_n| \phi_m + |\tilde{V}_m| \phi_n + \phi_n \phi_m) (g_{nm} \sin(\theta_n^\circ - \theta_m^\circ) - b_{nm} \cos(\theta_n^\circ - \theta_m^\circ)) \]
Warm-Start LP Approximation

- **Target part approximation**

\[
\hat{q}_{nm}^t = -|\tilde{V}_n|^2 b_{nm} + |\tilde{V}_n||\tilde{V}_m| b_{nm} \cos(\theta_n^o - \theta_m^o) - |\tilde{V}_n||\tilde{V}_m| g_{nm}(\theta_n^o - \theta_m^o)
\]

- **Delta part approximation**

\[
\hat{q}_{nm}^\Delta = -|\tilde{V}_n| b_{nm} (\phi_n - \phi_m) - (|\tilde{V}_n| - |\tilde{V}_m|) b_{nm} \phi_n
\]
**Model 1** The Warm LPAC Model.

**Inputs:**
\[ \mathcal{PN} = \langle N, L, G, s \rangle \] - the power network
\[ |\vec{V}^t| \] - target voltage magnitudes
\[ cs \] - cosine approximation segment count

**Variables:**
\[ \theta_n^o \in (-\infty, \infty) \] - phase angle on bus \( n \) (radians)
\[ \phi_n \in (-|\vec{V}^t|, \infty) \] - voltage change on bus \( n \) (Volts p.u.)
\[ \widehat{\cos s_{nm}} \in (0, 1) \] - Approximation of \( \cos(\theta_n^o - \theta_m^o) \)

**Maximize:**
\[ \sum_{(n,m) \in L} \widehat{\cos s_{nm}} \] \hspace{3cm} (M1.1)

**Subject to:**
\[ \theta_s^o = 0, \phi_s = 0 \] \hspace{3cm} (M1.2)
\[ \phi_i = 0 \ \forall i \in G \] \hspace{3cm} (M1.3)
\[ p_n = \sum_{m \in N \setminus \{s\}} \hat{p}_{nm}^t \ \forall n \in N \setminus \{s\} \] \hspace{3cm} (M1.4)
\[ q_n = \sum_{m \in N \setminus \{s\}} \hat{q}_{nm}^t + \hat{q}_{nm}^\Delta \ \forall n \in N \setminus \{s\} \setminus G \] \hspace{3cm} (M1.5)
\[ \forall (n,m), (m,n) \in L \]
\[ \hat{p}_{nm}^t = |\vec{V}_n^t|^2 g_{nm} - \bar{V}_n^t |\vec{V}_m^t| (g_{nm} \widehat{\cos s_{nm}} + b_{nm} (\theta_n^o - \theta_m^o)) \] \hspace{3cm} (M1.6)
\[ \hat{q}_{nm}^t = - |\vec{V}_n^t|^2 b_{nm} - |\vec{V}_n^t| |\vec{V}_m^t| (g_{nm} (\theta_n^o - \theta_m^o) - b_{nm} \widehat{\cos s_{nm}}) \] \hspace{3cm} (M1.7)
\[ \text{PWL}\langle \cos \rangle (\widehat{\cos s_{nm}}, (\theta_n^o - \theta_m^o), -\pi/3, \pi/3, cs) \] \hspace{3cm} (M1.8)
\[ \hat{q}_{nm}^\Delta = - |\vec{V}_n^t| b_{nm} (\phi_n - \phi_m) - (|\vec{V}_n^t| - |\vec{V}_m^t|) b_{nm} \phi_n \] \hspace{3cm} (M1.9)
Cold-Start LP Approximation

• Simply use the warm-start model with
  • Target voltages at 1.0
  • Use an appropriate $\phi$ for voltage-controlled generators

• Note that the delta part of reactive power becomes

$$\hat{q}_{nm}^\Delta = -b_{nm}(\phi_n - \phi_m)$$
Extensions of the LPAC Model

- **Range for generators**
  - Simply include a decision variable

- **Removing the slack bus**
  - No need for a slack bus in the LPAC model

- **Shedding load**
  - Simply use decision variables for loads

- **Additional constraints**
  - **Voltages:** \( |V| \leq |V_n^t| + \phi_n \quad \forall n \in N \)
  - **Apparent power:** \( (\hat{P}_{nm}^t)^2 + (\hat{q}_{nm}^t + \hat{q}_{nm}^\Delta)^2 \leq |S_{nm}|^2 \)
  - **Reactive power:** \( \sum_{m \in N} \hat{q}_{nm}^t + \hat{q}_{nm}^\Delta \leq q_n \quad \forall n \in G \)
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Experimental Results

- Wide variety of IEEE and MATPOWER Benchmarks
  - ieee14, mp24, ieee30, mp30, mp39, ieee57, ieee118, ieedd17, mp300
  - Small benchmarks are easy in general
  - IEEE 118 is also easy
  - All LPAC models solved almost instantly (LPs)

- This talk
  - MP300 for scalability and brevity

- Comparison with an AC Solver
  - LDC and LPAC solutions versus an AC solution

- Comparison with alternative linearizations
  - Evaluating the importance of all components
Line Active Power

Line Active Power Correlation (MW)

AC Power Flow

LDC Model

Cold-start LPAC Model
## Line Active Power

### Table 1: Active Power Flow Accuracy Comparison

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<thead>
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<th>Benchmark</th>
<th>Corr</th>
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<th>$\mu(\delta)$</th>
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Bus Angles

LDC Model

Cold-Start LPAC Model
Line Reactive Power

Cold-Start LPAC Model

Warm-Start LPAC Model
Bus Voltages

Bus Voltage Correlation (Volts p.u.)

Cold-Start LPAC Model

Warm-Start LPAC Model
Cosine Approximation

Quality of a Piece–wise Linear Cosine Approximation

Radians

-0.4 -0.2 0.0 0.2 0.4

0.93 0.95 0.97 0.99

- \cos(x) \quad \text{pwl-\cos}(x)

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Importance of $g$: Reactive Power

Cold-Start LPAC Model ($g=0$)
Importance of g: Bus Voltages

Cold-Start LPAC Model (g=0)  Cold-Start LPAC Model
Importance of $\cos$: Reactive Power

**Cold-Start LPAC Model ($\cos=1$)**

**Cold-Start LPAC Model**
Importance of cos: Bus Voltages

Cold-Start LPAC Model (cos=1)
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  • Build on top of the cold LPAC model
• Power Restoration
Capacitor Placement

• The Problem
  • place capacitors in a power network to improve voltage stability

• Minimize the number of capacitors subject to
  • lower bounds on the voltages
  • upper bounds on reactive capacitor injection
  • upper bounds on reactive generation injection
Capacitor Placement

Inputs:
- $\bar{q}_n^g$ - injection bound for generator $n$
- $q^c$ - capacitor injection bound
- $|\bar{V}|$ - minimum desired voltage magnitude

Inputs from The Cold LPAC Model

Variables:
- $q^c_n \in (0, \bar{q}^c)$ - capacitor reactive injection
- $c_n \in \{0, 1\}$ - capacitor placement indicator

Variables from The Cold LPAC Model

Minimize:
$$\sum_{n \in N} c_n$$

Subject to:
- $|\bar{V}| \leq 1.0 + \phi_n \leq 1.05 \quad \forall n \in N$
- $q^c_n \leq Mc_n$
- $q_n \leq \bar{q}_n^g \quad \forall n \in G$
- $q_n = \sum_{n \neq m}^{n} \hat{q}_{nm} + \hat{q}_{nm}^\Delta \quad \forall n \in G$
- $q_n = \sum_{n \neq m}^{n} \hat{q}_{nm} + \hat{q}_{nm}^\Delta \quad \forall n \in N : n \neq s \land n \notin G$

Constraints from The Cold LPAC Model
Experimental Results

• Modified IEEE 57 Benchmark
  • Remove the transformers
  • Remove the synchronous condensers
  • This induces severe voltage problems
  • Impose increasingly tighter voltage lower bounds

• The capacitor placement model
  • Meets all voltage requirements but is an approximation

How well does it do compared to the AC model?
IEEE57: 0 Capacitor
Table 1: Capacitor Placement: Effects of $|\tilde{V}|$ on IEEE57-C, $\overline{q^c} = 30$ MVar

| $|\tilde{V}|$ | min($|\tilde{V}|$) | max($|\tilde{V}|$) | max($q_n$) | $\sum c_n$ | Time (sec.) |
|------------|----------------|----------------|-----------|-----------|-------------|
| 0.8850     | 0.000000       | 0.0            | 0.0       | 1         | 1           |
| 0.9350     | 0.000000       | 0.0            | 0.0       | 3         | 8           |
| 0.9600     | 0.000000       | 0.0            | 0.0       | 5         | 156         |
| 0.9750     | -0.000000      | 0.0            | 0.0       | 6         | 177         |
| 0.9775     | -0.000000      | 0.0            | 0.0       | 6         | 139         |
| 0.9800     | -0.000000      | 0.0            | 0.0       | 6         | 75          |
| 0.9840     | -0.000802      | 0.0            | 0.0       | 7         | 340         |
.9600
Bus Voltage Correlation (Volts p.u.)
Bus Voltage Correlation (Volts p.u.)

LL-LDC Power Flow

AC Power Flow
Bus Voltage Correlation (Volts p.u.)

LL-LDC Power Flow

AC Power Flow
Bus Voltage Correlation (Volts p.u.)

AC Power Flow

LL-LDC Power Flow

0.96 0.98 1.00 1.02 1.04

0.96 0.98 1.00 1.02 1.04
Bus Voltage Correlation (Volts p.u.)
Bus Voltage Correlation (Volts p.u.)

AC Power Flow

LL−LDC Power Flow
Outline

• Motivation
• The LPAC Model
• Experimental Results
  • LDC versus LPAC versus AC solutions
  • LPAC Variants
• Capacitor Placement Problem
  • Build on top of the cold LPAC model
• Power Restoration
  • Build on top of the warm LPAC model
Power Restoration

• See Carleton’s talk on Thursday 5:40 – 6:00
  • Just want to make you curious here
Demand Maximization

**Inputs:**
- $p_n^g$ - maximum active injection for bus $n$
- $p_n^l$ - desired active load at bus $n$
- $q_n^l$ - desired reactive load at bus $n$

Inputs from the Warm LPAC Model

**Variables:**
- $p_n^g \in (0, p_n^g)$ - active generation at bus $n$
- $q_n^g \in (-\infty, \infty)$ - reactive generation at bus $n$
- $l_n \in (0, 1)$ - percentage of load served at bus $n$

Variables from the Warm LPAC Model

**Maximize:**
$$\sum_{n \in N} l_n$$

**Subject to:**
- $p_n = -p_n^l l_n + p_n^g \quad \forall n \in N$
- $q_n = -q_n^l l_n + q_n^g \quad \forall n \in N$
- $q_n^g = 0 \quad \forall n \in N \setminus G$
- $n \neq m$
- $q_n = \sum_{m \in N} \hat{q}_{nm}^t + \hat{q}_{nm}^\Delta \quad \forall n \in G$

Constraints from the Warm LPAC Model
# IEEE-30 Contingencies

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Power Restoration

AC Restoration Timeline

- Restoration Action
- AC Power Flow (MW)

LDC–ROP
Power Restoration

DC Restoration Timeline

DC Power Flow (MW)

Restoration Action

LDC–ROP
LPAC–ROP
Power Restoration

AC Restoration Timeline

AC Power Flow (MW)

Restoration Action

LDC–ROP

LPAC–ROP

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From imagination to impact
Power Restoration

AC Line Overloads

Cumulative Overload (MVA)

Restoration Action

LDC
LPAC+R
LPAC+R+V
Power Restoration

AC Voltage Stability

Cumulative Instability (Volts p.u.)

Restoration Action

LDC
LPAC+R
LPAC+R+V
Conclusion

- **LPAC Models: Linear-Programming approximations**
  - Much more accurate than the LDC model
  - useful outside normal operating conditions
  - reason about voltage magnitudes and reactive power
  - can be embedded in MIP solvers

- **Experimental results**
  - Very high accuracy when compared to AC solutions

- **Case studies**
  - Capacitor placement problem
  - Power Restoration